Towards Flipped Learning in Upper Secondary Mathematics Education

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Abstract

Challenges for students in the 21st century, such as acquiring technology, problem-solving, and cooperation skills, also necessitate changes in mathematics education to be able to respond to changing educational needs. One way to respond to these challenges is by utilizing recent educational innovations in schools, for instance, among others are flipped learning (FL) approaches. In this paper, we outline our explorative educational experiment that investigates vital elements of mathematics learning in FL approaches in upper secondary education. We describe the methodologies and findings of our qualitative study based on design-based research to discover key elements of FL approaches in upper secondary education. Analyzing the oral and written data collected over ten months using grounded theory approaches suggested categories (a) confidence when learning; (b) learning by working, and, and (c) flexibility when learning could be essential to understand FL approaches practices in mathematics classrooms. These categories indicate that when using FL approaches in mathematics learning, it could be essential for students to acquire knowledge in a confident and adaptable environment actively.

Keywords: flipped learning, mathematics education, student-centered learning

A. Introduction

Flipped learning (FL) approaches can be characterized by a flexible educational environment, a new learning culture, intentional content, and a well-prepared professional educator (Flipped Learning Network, 2014). To facilitate changes in schools, Fadel (2008) suggested the frame of 21st-century skills: (a) learning and innovation skills (e.g., critical thinking, problem-solving or creativity); (b) information, media and technology skills; and (c) life and career skills (e.g.,
flexibility, responsibility or adaptability), which students should acquire. FL approaches could offer educational approaches where these 21st-century skills are fostered in mathematics classrooms. Education following FL approaches can be interpreted as a further development of flipped classroom (FC) education (Flipped Learning Network, 2014). In FL, students’ interests and preferences treated even more centrally than in FC education. In addition, learning with FL approaches does not lead to a division of acquiring knowledge (before lessons) and utilizing knowledge (in lessons) as it is associated with FC methods (Weinhandl & Laviczka, 2018). In FL, students can decide the direction and pace of knowledge acquisition and its applications. With this interplay between acquisition and application, students can switch between individual and group learning in study situations (Flipped Learning Network, 2014). Thus, FL approaches’ intense focus on students’ interests and preferences, integrated into the learning environment could further facilitate their learning and innovation; information, media and technology; and life and career compared to practices carried out through FC methods.

In our paper, we illustrate how mathematics education based on FL approaches could be designed to address the development of these 21st-century skills. As flipped learning is a further development of FC education, it could be fruitful to scientifically investigate this new way of teaching flipped in real and everyday situations. When exploring mathematics education following FL approaches, we focus on central elements and requirements of FL designs for secondary school education. By focusing on the elements of FL designs, it should be possible to deduce theories to which mathematics education with FL approaches could be integrated into education in schools.

To illustrate how FL mathematics education could contribute to achieving 21st-century skills, FC and FL approaches are described in more detail in the next section. In the section "Theoretical Background," we outline elements of (social) constructivism as well as a seamless learning theory that is essential for FC and FL education. As a result of our explorative educational experiments, we concluded how core categories such as (a) confidence when learning, (b) learning by working, and (c) flexibility when learning typical for learning mathematics with FL approaches could be characterized.

1. Flipped Classroom

As FL approaches can be understood as a further development of FC education (Flipped Learning Network, 2014), we start by outlining the cornerstones of learning and teaching with FC approaches followed by characteristics of FL. FC approaches could be described as an educational approach where direct instruction and passive learning happens outside of classrooms. In-class time is used for active and student-centered learning (Maciejewski, 2015; Wasserman, Quint, Norris, & Carr, 2015). Consequently, lower levels of cognitive work (Krathwohl, 2002) are tackled before class, and in-class time is filled with higher levels of cognitive work (Galway, Corbett, Takaro, Tairyan, & Frank, 2014). Long et al. (2016) defined FC as an environment where students’ learning happens at home while class time is reserved for exercises and practical activities and, consequently, gives students more personal responsibility in their education. The descriptions and definitions of an FC above illustrate that, despite specific differences, the central element of an FC is that simple learning activities are done at home, and then more complex learning and applications of knowledge happen in classrooms. If education following FC approaches is linked to 21st-century skills, it can be deduced that following this approach of education, the in-class phases of learning are particularly suitable for developing 21st-century skills. However, to develop a more holistic synthesis of school learning and developing 21st-century skills, we have focused our explorative educational study on mathematics learning using a flipped learning approach.

2. Flipped Learning

According to the Flipped Learning Network (2014), FL can be seen as a further development of FC approaches. Since Cronhjort, Filipsson, & Weurlander (2017) and Maciejewski (2015) could demonstrate in their studies that mathematics education following FC approaches could positively affect students’ learning performances and motivation, it could be fruitful implementing a further development of this promising approach into teaching and learning mathematics. However, FC does not necessarily lead to FL. To counter misconceptions, the Flipped Learning Network proposed the following definition of Flipped Learning.
“Flipped Learning is a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter.” (Flipped Learning Network, 2014)

Additionally, the Flipped Learning Network (2014) proposed four pillars of FL described as follows:

- **Flexible environment:** To support FL, there is a need for a flexible environment, where students can choose where and when they learn.
- **Learning culture:** There should be a shift from a teacher-centered to a learner-centered model. Students should be allowed to engage in meaningful activities for constructing knowledge actively without teachers being central.
- **Intentional content:** In FL environments, teachers should choose wisely what learning content should be implemented in the self-learning space so that the time gained can be used for student-centered activities.
- **Professional educator:** As professional educators, teachers should provide students with feedback when needed, provide activities, and connect with others to reflect and further develop their teaching. Hence, the teacher role in FL environments can be challenging, even more than in traditional classroom settings.

According to the description of FL based education, students have more power to co-determine with the teacher what to work on compared to FC approaches. Students' increased power to co-determine education is reflected in students' ability to adapt to the learning environment, learning actions, and the content and the issues emerging in classrooms. This increased opportunities to co-determine education should also positively affect students' learning and innovation skills or life and career skills. Consequently, it could be mathematics education following an FL approach that facilitates students to acquire 21st-century skills in school learning.

3. **Flipped Learning approaches and mathematics education**

Education with FL approaches, as described above, is characterized among other aspects by a flexible environment, a new learning culture, and intentional content. A flexible environment, a new learning culture, and intentional content could make an FL environment a positive place. A positive learning environment could be especially important in mathematics education, as mathematics causes anxiety for many students. Offering such a positive learning environment, this anxiety could be reduced, which should positively affect student performance (Hung et al., 2014; Lee & Johnston-Wilder, 2013). According to Chao et al. (2016), a positive learning environment should also increase student motivation, which should be a vital component for successful mathematics teaching.

A flexible learning environment – i.e., students can decide for themselves when, where, and how to learn – when learning following flipped learning approaches should positively impact students' self-efficacy and confidence. Students' self-efficacy and confidence are also vital in mathematics education, and successes when learning mathematics and students not afraid of making mistakes could foster students' self-efficacy and confidence (Burton, 2004; Chao et al., 2016).

Changes in a learning culture and intentional content in FL education could also ensure that questions and content relevant to students are addressed in class. According to Gainsburg (2008) and Hodges & Hodge (2017), questions, and content relevant to students characterize good mathematics education. Questions and content relevant for students also mean that real-world problems and tasks are often tackled in mathematics education. According to Elbers (2003), mathematics education should always focus on meaningful and real problems. Addressing significant and real problems also means addressing open (Coles & Brown, 2016) or non-routine (Tait-McCutcheon & Loveridge, 2016) problems in mathematics education, which could improve mathematics education.

Tackling intentional content as well as meaningful and real problems when learning mathematics also means that mathematics education is closely linked to the respective society. According to Samuelsson (2006), every school is also part of society, and those changes that affect a society also have an impact on schools. Recently, few changes have had such a substantial impact on societies and, therefore, on schools as technological innovations. According to Chao et al. (2016) and Fogarty et al. (2001), many students appreciate using technologies when learning mathematics, as utilizing technologies could facilitate to make students' learning achievements
more concrete. However, Samuelsson (2006) stresses that merely using technologies in education is not yet learning with technologies. Decisive for the success of learning with technologies should be how technologies are utilized, whereby it is recommended to employ technologies for discovering and exploring mathematics.

Using technologies, intentional content, flexible learning environment, and the changing learning culture, when education according to FL approaches addresses significant and real problems, could result in learning is carried out with a partner or in a group. According to many mathematics education experts (e.g., Bell & Pape, 2012; Elbers, 2003; Lee & Johnston-Wilder, 2013), also doing and learning mathematics should be interpreted as a social process.

Describing FL approaches and comparing learning according to FL principles and learning mathematics indicates that a synthesis of FL approaches and mathematics education could be quickly established. However, investigating FL mathematics education and FC education also demonstrates that FC education and FL education consist of different elements and reflect different approaches to learning. These differences are not yet present in the current literature on mathematics education, since some authors use flipped classroom and flipped learning synonymously (e.g. González-Gómez et al., 2016; J. Lee, Lim, & Kim, 2017) or elements of flipped learning occur in research but are described as a flipped classroom (Weidlich & Spannagel, 2014).

According to this gap in research, our explorative educational experiment aims to identify how mathematics education at a secondary level could be designed according to the principles of FL approaches and exploring opportunities and risks when learning mathematics following FL approaches at a secondary level.

B. Literature Review
In the theoretical background of our paper, epistemologies and learning theories are illustrated, which could be assigned to learning following flipped approaches. By discussing epistemologies and learning theories in FL approaches, we aim to highlight those elements of a learning process that are characteristic of flipped approaches. Since flipped approaches merge with many epistemologies and learning theories, we will focus only on those epistemologies and learning theories in the theoretical background that share many common features with an FL approaches and are relevant to our research aim – namely (social) constructivism and seamless learning.

1. Constructivism
A characteristic of FL mathematics education is that students could choose learning materials, learning environments, and social forms (i.e., individual work, partner work, or group work) themselves. By applying new mathematical concepts, students should increase their body of knowledge themselves. Consequently, learning mathematics following FL approaches, as one method of an active-constructive development of mathematical knowledge, and constructivism or more precise constructivist learning theories and their anti-representationalism characteristics could have much in common.

Constructivism in learning theory could be described as a combination of several approaches since the beginning of the 20th century whereby Dewey, Piaget, Kelly, or Vygotsky are named as decisive personalities in this process. Despite or precisely because of this long history, constructivism could not be uniformly defined or described in current literature, but it is ascribed to most interpretations of constructivism that they have a common ground (Duit, 1995; Gräsel et al., 1997; Richardson, 2003):

Knowledge created through hands-on activities, new knowledge depending on previous experience and existing knowledge, and new knowledge is embedded in an existing construct of experience and knowledge are central elements of constructivism according to Euler (2001), Gräsel, et al., (1997) and Koohang et al., (2009). With the thesis knowledge as a construction von Glasersfeld (1995) simplifies Piaget's explanation of learning, Kerres & De Witt (2004) and Koohang et al. (2009) stress that when learning according to a constructivist approach, it would be important that learning processes are triggered by real-world issues that could be given or determined by learners. Treating real and, therefore, complex problems should lead to multiple perspectives on new knowledge. According to these explanations and key elements of learning according to constructivism, the following formula could be deduced:

Learning according to constructivism = Previous Knowledge + New Information + Cooperative and Social Interactions + Applications
(Bentley et al., 2007; Denton, 2012; Richardson, 2003)
Especially when comparing the constructivism equation and learning mathematics following flipped approaches indicates that constructivism and FL have numerous common elements. Even aspects of social constructivism (Vygotsky & Cole, 1978) that emphasize the social environments in which learners generate their knowledge are addressed in an FLA.

Our explorative flipped learning experiment aims to integrate constructivist elements of FL mathematics education, specifically into secondary mathematics education, and to investigate which elements of this educational approach could be essential and promising when learning mathematics.

2. Seamless learning theory
Elements of seamless learning theory could also be found in FL approaches as students utilize different learning materials in individual or group learning spaces according to their needs.

Wong (2015) adapted Sharples et al.’s (2012) exposition and defined seamless learning as "when a person experiences continuity of learning, and consciously bridges the multifaceted learning efforts, across a combination of locations, times, technologies or social settings." Moreover, Wong (2012) stated that mobile technologies enabled learners to learn across different settings and called such a learning approach "seamless mobile learning."

Hwang et al. (2015) introduced seamlessly flipped learning by using information and communication technology to seamlessly connect the in-class phase with at-home learning activities to support the continuous flow of learning. They proposed several principles and strategies for the development of seamless FL scenarios. The principles fitting our experiment were considered in the lesson planning. For instance, we designed in-field activities where students had to apply their knowledge to real-world problems.

3. Research questions
According to our research goal – identifying how mathematics education at a secondary level could be designed following FL approaches – and the theoretical background of flipped education, the following research questions arise for our experiment: (1) what elements of FL approaches are relevant for students when mathematics is learned following FL approaches?; and (2) what activities do students need to perform, and what roles should students assume when mathematics is learned following the principles of FL approaches in secondary education?

To investigate the research questions, we have conducted an explorative design-based FL experiment at two schools over ten months. Next, we present the characteristics and design of our FL experiment, as well as how the FL design has changed in the course of research.

C. Method
To address our research questions, we conducted an educational study with two classes of an upper secondary school. In this study, we collected and evaluated data according to the principles of design-based research and grounded theory approaches.

1. Lesson description
Our FL experiment was conducted with a total of five classes in two schools – an urban college of business administration and a grammar school. Altogether, more than 130 students and four teachers were involved in our FL experiment. Students of our FL experiment were from the 9th and 10th grade and, therefore, from 14 to 17 years old. The topics that were covered in our FL experiment included all subject areas of the mathematics curriculum.

One characteristic of our FL experiment was that modern technologies were used intensively when teaching and learning mathematics. Utilizing modern technologies means that students could use tablets, notebooks, or other personal devices throughout the entire period of our educational experiment. In terms of software, in our FL experiment, the ePortfolio software Mahara and the learning management system Moodle were used to communicate assignments and learning materials, but also for students to be able to present and share their learning outcomes. The mathematical software package GeoGebra was utilized in our FL experiment to model real-world situations mathematically and solve mathematical problems. However, throughout the entire FL experiment, students were also free to use other technologies if these resources were conducive to learning from a student perspective. It was the students’ decision whether to work individually or in groups to achieve the learning goals. Irrespective of the social
form chosen, all students had to document their learning progress individually, solve tasks, and create learning products.

In order to achieve our research goal and to integrate findings from the research process into our experiment, our FL experiment was divided into four phases:

Table 1. Description of the phases of our flipped learning experiment

<table>
<thead>
<tr>
<th>Phase</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>Learning materials and tasks are made available to the students at the</td>
</tr>
<tr>
<td></td>
<td>beginning of the sequence. Students can orchestrate the learning process</td>
</tr>
<tr>
<td></td>
<td>for two weeks.</td>
</tr>
<tr>
<td>Phase 2</td>
<td>Learning materials and tasks are made available to the students at the</td>
</tr>
<tr>
<td></td>
<td>beginning of the sequence. The learning process is documented by the</td>
</tr>
<tr>
<td></td>
<td>students on their Portfolio page (Mahara) and shared with the class.</td>
</tr>
<tr>
<td>Phase 3</td>
<td>In addition to the learning activities and conditions of Phase 2, there is</td>
</tr>
<tr>
<td></td>
<td>a more detailed subdivision of the learning sequences and classwork times.</td>
</tr>
<tr>
<td></td>
<td>Furthermore, Q&amp;A phases were integrated.</td>
</tr>
<tr>
<td>Phase 4</td>
<td>In addition to the learning activities and conditions of Phase 3, there are</td>
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<tr>
<td></td>
<td>teaching units in which students can work in different learning</td>
</tr>
<tr>
<td></td>
<td>environments (classroom or computer lab), and in each learning</td>
</tr>
<tr>
<td></td>
<td>environment, a teacher is available to students.</td>
</tr>
</tbody>
</table>

As our study aims to discover new insights through our research and identify which elements of FL approaches could support learning mathematics in upper secondary schools, we utilized a mix of qualitative research approaches to data collection and analysis. This mix of qualitative research approaches consists of design-based research (DBR) – mainly to discover and further develop crucial elements of an FL design – and grounded theory approaches to develop theories on mathematics learning following FL approaches.

2. Design-based research

In our explorative study, we aimed to explore how education following FL approaches should be designed to facilitate students in developing 21st-century skills. Additionally, we aimed to investigate which elements of FL approaches could be relevant for students when learning mathematics and which activities students should perform when learning mathematics following FL approaches. According to these research goals and research questions, design-based research should be an appropriate research methodology. Design-based research is "a systematic but flexible methodology aimed to improve educational practices through iterative analysis, design, development, and implementation, based on collaboration among researchers and practitioners in real-world settings, and leading to contextually sensitive design principles and theories." (Wang & Hannafin, 2005, pp. 6-7) The interplay of research and practice is characteristic of DBR approaches. This exchange should cover the entire state of research. Therefore, both research and practice should be oriented towards this research approach and benefit from it (Anderson & Shattuck, 2012; Cobb et al., 2003). In our explorative study, this synthesis of theory and practice was established by researchers and practitioners (teachers) working closely together over the entire research period. Close collaboration between researchers and practitioners means that design development, design application, and design analysis have always been performed together. Reinmann (2005) has emphasized the difficulty of creating laboratory conditions in educational science as learning and teaching are too complicated and have justified the necessity of applying DBR in education research.

In most cases, the starting point of a DBR is an initial problem in an educational context, followed by a literature search concerning this issue (Anderson & Shattuck, 2012). Since we want to develop a flipped learning environment further and generate scientific findings during further development, this learning environment must be applied under real conditions. Based on scientific observations and analyses, the learning environment will be adapted and then re-implemented because Zheng (2015) has stated that several design cycles are required to obtain scientific outcomes. Finally, the effects of different designs are compared, and the strengths and weaknesses of the different interventions are identified.
3. Grounded theory approaches
Since our research aim is exploring crucial elements when learning mathematics following FL approaches, we have collected and evaluated different data according to grounded theory approaches (GTA). Because one of GTA's main goals is to gain new insights and understandings of reality and people in real environments as well as activities performed within such environments, GTA is well suited to bring us closer to our research goal and questions (Charmaz, 2006; Glaser & Strauss, 1999; Strübing, 2004; Woods et al., 2016). Another reason for choosing GTA is that these approaches of research aim to investigate social and professional networks and to shed light on human activities in these networks. By cyclically investigating such networks, GTA could transfer practical knowledge into theory (Glaser & Strauss, 1999; Kamin, 2013; Mey & Mruck, 2011). The fact that it is in the nature of GTA that specific aspects and small groups of society are examined, but that this investigation is carried out in-depth, has also led us to opt for GTA (Rosenkranz, 2017; Strübing, 2004).

In our research, we follow an interpretative understanding of GTA, according to Charmaz (2006). Breuer et al. (2009) state that researchers are decisive factors in GTA, making direct contact with the field under study and its members, and researchers could also become part of the research field. In this close contact with the research field, researchers have to assume two roles: They become part of the research field and remain part of the scientific world. In order to better meet these two roles of researchers in a GTA, we have decided that one researcher should also teach in our FL experiment, and a second researcher should take on a scientific meta-level.

According to Charmaz (2006), it makes a difference in who collects data and what research tools are utilized when collecting data. In order to get as deep insights into our FL experiment as possible, we decided that both researchers should collect data and that written feedback (more than 250), in-depth individual interviews with students and teachers (17) and group interviews (2) should be conducted over the entire research period (May 2018 to February 2019).

Throughout the research period, the data collected were independently coded (open, axial and selective) by the researchers, constantly compared with existing data (from our research and current literature), included in category and concept development and memo writing as well as used for further data collections (see Table 2 and Figure 1).

D. Findings
1. Findings
To investigate which elements of FL approaches could be relevant for students when learning mathematics and which activities students should perform when learning mathematics following FL approaches, a close collaboration between researchers and practitioners was established in our explorative study. Throughout our flipped learning experiment, all researchers and teachers involved wrote notes after each lesson. The aim of writing lesson notes was, on the one hand, capturing impressions of teachers. By recording teachers' impressions, a temporal comparison before and after educational interventions should be facilitated. In this context, we defined the term educational interventions as incorporating new elements into our FL design or the modification and adaptation of existing elements of our FL design. On the other hand, lesson notes should enable comparing the inside view (of the teacher) and the external view (of the observing teacher or researchers) of lessons. By synthesizing these two perspectives on our FL experiment, findings on essential FL design elements should be possible to improve.

After each educational intervention, we collected written and verbal (interviews) feedback from the students and wrote memos. First, students' feedback was read or listened to individually several times. Reading or listening to the feedback aimed to provide an initial overview and a rough structure of the new data. After obtaining a rough structure of the new data, the feedback was individually and openly coded by the researchers. Coding the data using open coding techniques aimed to break up the newly collected data and to detect first units of meaning. After each of us had finished open coding, individual codes were compared, discussed, and, if possible, grouped. This grouping resulted in a total of 67 open codes (see Table 2, left column). Then, these open codes were reapplied to the existing data, resulting in 18 codes of a higher level of abstraction (see Table 2, middle column).
Table 2. Development of core categories formation. The complete table of the category formation process is available on the following link: Link

<table>
<thead>
<tr>
<th>Open codes</th>
<th>Open codes of a higher level of abstraction</th>
<th>Core categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>alternative assessment</td>
<td>joint development of knowledge</td>
<td>confidence when learning</td>
</tr>
<tr>
<td>requirements through complexity</td>
<td>confidence in the learning process</td>
<td>learning by working</td>
</tr>
<tr>
<td>applying knowledge</td>
<td>transparency in the assessment process</td>
<td>flexibility when learning</td>
</tr>
<tr>
<td>working</td>
<td>learning media as a support</td>
<td></td>
</tr>
<tr>
<td>working in class</td>
<td>developing learning outcomes individually</td>
<td></td>
</tr>
<tr>
<td>task design and explanation</td>
<td>individual time management in the learning process</td>
<td></td>
</tr>
<tr>
<td>quantity of tasks</td>
<td>hands-on learning approach</td>
<td></td>
</tr>
<tr>
<td>information</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>grading</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>relation to the real world</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>sense of achievement</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Next, these 18 codes of a higher level of abstraction were individually axially coded (see Figure 1). In axial coding, we placed different codes of a higher level of abstraction (phenomenon) at the centre of our investigation. Around this phenomenon, we arranged further corresponding codes of a higher level of abstraction following the tripartite division **causes** – **action strategies** – **consequences**. For a further abstraction of our research data, the codes of a higher abstraction level of the action strategies were further investigated and compared.

![Figure 1. The prototypical procedure of axial coding](image)

After comparing our results with the axial codes, we were able to identify nine primary categories for our flipped learning experiment. By reapplying these nine categories to the existing data and by further selective codes, we were finally able to deduce three core categories – namely (a) confidence when learning, (b) learning by working, and (c) flexibility when learning.

The following quotations prototypical for the core categories were translated by the authors from German into English. The identifier WF represents students' written feedback, and the identifier VF represents students' verbal feedback.

1.1 Confidence when learning
As described earlier, in FL mathematics education that students can determine their learning paths, select real-world problems to be tackled, and design learning materials and social interactions by themselves. This high degree of freedom and self-determination of students in FL environments could lead to active and self-responsible learning in classrooms. For active and self-responsible learning to be as friction-free as possible, it may be relevant for students to feel confident in the learning process. However, confidence in an FL environment is not a one-dimensional phenomenon but influenced by factors such as tasks offered by teachers, learning materials, learning environment setup, and constant feedback.
Students' feedback indicates that it is vital to feeling confident about the subject matter before starting to work and apply knowledge. Being confident in the subject matter means that students want to understand in general or, in theory, what the principles of a mathematical concept are.

VF: When we begin a topic, that he [teacher] explains it, so everyone has the basis for the basic knowledge and that everyone can start learning with this basic knowledge.

It is also crucial for students to have confidence in the practical aspects of a mathematical concept and know-how to perform certain mathematical operations. Practical and operational confidence should be achieved by processing and solving prototypical examples according to student data.

VF: I would be in favor if we as a whole class [students and teacher] would briefly study the topic for one or two lessons and work on a few examples before we [students] start solving examples ourselves.

Teachers' confidence role is to provide a protected learning environment and sufficient time for students to gain confidence and self-reliance in a new mathematical topic and thus to start a work process or work phase positively.

WF: After the "basics" of the subject matter has been explained, it is easier to solve examples ourselves without getting confused.

If students cannot achieve such a level of confidence before a work and application phase starts, student feedback demonstrates that feelings of “fear” or “not having understood it” could be an emotional barrier. This barrier could make learning mathematics difficult or even impossible.

VF: So far, everything is going well, but if someone has difficulties in mathematics, then he has no chance at all because somehow, you do not get enough explanation from the teacher. So if you ask, of course, everything will be explained, but [...] so for me, it is barely ok.

However, it is not only the teachers who provide confidence but also learning materials that are vital in the confidence category. The feedback data showed that learning materials are expected to be an anchor when learning about or applying mathematical concepts. An anchor in the learning process means that learning materials should provide assistance whenever there are problems or ambiguities in a learning process.

VF: I find these videos very practical, which he [teacher] always uploads for specific topics.

Similar requirements placed on learning materials concerning confidence, particularly, also apply to FL environments in general. According to students' feedback, learning environments could provide confidence if students know that a learning environment could be adapted so that each student could use appropriate elements of the learning environment to achieve their aims.

WF: The separate lessons on Wednesday were an advantage for me. I could continue to work on the computer without any problems, but I could also continue working in groups in the classroom.

Concerning confidence, regular feedback during work phases is also crucial for students. Individual feedback at regular intervals should prevent students from feeling that they are working blind, which could have a negative impact on confidence. Likewise, regular feedback could lead to regular success, which should have a positive effect on student motivation and, thus, on student confidence.

VF: I also think it's good that we do weekly revisions because you know where you stand right now and what you could perhaps do even better. These revisions are useful.

1.2. Learning by working
Evaluating the feedback data indicates that tackling mathematics in an FL environment is interpreted by students as work. Students describe activities in an FL environment more often as doing or working than learning. The fact that doing mathematics following FL approaches could be demanding is recognized and described by both students and teachers.

VF: It [working in an FL setting] is not more. It's just in a different approach, and it's just a little more difficult for me and therefore, a little more intense for me.

Although mathematics education in an FL environment and the associated work of students is described as being intense, students' feedback illustrates that students want to work when learning mathematics. Increasing work also leads students to describe themselves as being more active in FL education than in traditional mathematics teaching or in other subjects.

VF: So [we are given] more work orders and so on – and you do more, you are more active. So I am more active in mathematics lessons than in German lessons.
Students state that by working in mathematics education and doing mathematics, they engage more intensively with the topics being dealt with and learn more through this intensive engagement. Students also report that they can develop mathematical content on their own, making it easier to learn mathematics and forcing them to understand the subject matter in a positive sense.

VF: The fact that we work a lot independently is helpful. [...] The fact that you have to work all this out yourself leads to the fact that you usually understand it anyway and are almost forced in a positive sense to be able to do it and to learn it even if you do not understand it.

Another point that was mentioned positively by students in connection with working in FL mathematics education is that doing mathematics often leads to a concrete learning artifact at the end of a learning sequence. Regarding concrete learning artifacts, students explain that they are proud of their work and the learning artifact associated with their learning progress.

WF: In the end, you are always so proud when you see your [Mahara] page.

When doing mathematics and thus creating mathematics learning artifacts, it is crucial to students that tailored feedback is provided for the individual work steps and, thus, the learning process. The importance of tailored feedback for students is particularly evident in those learning units in which education was offered in two learning environments, each with a teacher. As a certain feature of this educational setting, the students state that just-in-time feedback was possible and that working and thus, the learning process was not interrupted by waiting for the teacher or feedback.

WF: I also think it’s good that we could split up [classroom or computer lab] because you can ask more questions, and it’s not so loud.

Although working in FL mathematics education is described by students as strenuous but positive, student feedback also reflects that the number and intensity of work orders should be well dosed since too many work orders could lead to excessive demands or confusion.

VF: It’s ok, but it’s way too much. We really have to do a lot of things and as I said before there are a lot of work orders and we don’t know when we have to submit them and then we get confused and we don’t know what kind of topic we are dealing with at that moment.

1.3. Flexibility when learning
In accordance with current literature (Flipped Learning Network, 2014), students’ feedback indicates that flexibility is a crucial element when education follows FL approaches. Similar to the descriptions of the Flipped Learning Network (2014), flexibility in our FL experiment is recognized and positively described by the students in the areas of time, place, social form, or learning materials.

WF: I actually think it’s good to be able to decide what you do now or how much time you spend on one topic.

VF: Well, I can organize it [learning process] well, and yes, it is also much more practical than always working together as a class because everyone needs a different amount of time for certain exercises.

Besides these general aspects of flexibility, students also state flexibility concerning mathematics in general. Flexibility in mathematics means that the students describe it as positive that in an FL approach to mathematics education, one could take enough time to look at all the solutions and then choose the best approach individually.

WF: In addition, one could “get to know” several different calculation methods and could thus choose the “simplest way.”

Flexibility in an FL approach to mathematics education also leads to students having more say in the educational process, which students describe as positive. In addition to more co-determination, flexibility in FL mathematics education could also lead to higher student responsibility in the learning process. More responsibility means that students also need more meta-competencies when learning mathematics with FL approaches. Students named time management, self-discipline, and teamwork as key meta-competencies for mathematics education with FL approaches.

VF: I think it [flipped learning] is better because you learn a bit of self-discipline and that you also have to decide for yourself: “Ok I have to do that now” and that you just learn to deal with it [responsibility] on your own.
This wealth of meta-competencies led some students to be challenged by the demands of FL mathematics education. This demand, or at times excessive demand, results in some students wanting more and more in-depth specifications from teachers regarding structure and time.

WF: I think it would be better if we could get the tasks in small parts so that you don’t have the possibility to always postpone them [tasks] until the last day.

Analyzing and evaluating the data collected during our FL experiment indicated that it could be vital for students to feel confident when they learn independently and self-responsibly, as it is one of the characteristics of FL approaches. Furthermore, it could be the flexibility that students appreciate when they learn with FL approaches. Flexibility when learning following FL approaches concerns, on the one hand, choosing the learning environment, the social form, and the learning materials, and on the other hand, selecting the basic approach to individual learning. Learning mathematics utilizing FL approaches has been described by learners as work. However, work was mostly interpreted positively, and many students appreciated having the opportunity to work on their mathematical body of knowledge independently.

2. Discussion
Our research aimed to investigate how mathematics education in secondary schools following FL approaches could be designed and what opportunities and risks are associated with FL mathematics education. When examining our research objective, it became evident that flexibility when learning could be of great importance. Flexibility when learning and teaching mathematics should also lead to a learning process or learning environment being more easily individualised. The individualization of learning processes or learning environments is equally essential for both FC (Tillmann et al., 2014) and FL education as well as mathematics education (Harkness & Stallworth, 2013, Lee & Johnston-Wilder, 2013). In our FL experiment, it could be demonstrated that individualization in education does not necessarily happen by itself. In order to achieve individualization in secondary education, various teacher activities and student meta-competencies are needed. Since framework conditions for individualization of teachers and students should be developed slowly, a slow approach at the beginning of FL education is recommended, as with FC education (e.g., García-Peñalvo et al., 2016; Muir & Geiger, 2016: Long, Cummins, & Waugh, 2016) – i.e., an FL evolution and not an FL revolution.

According to numerous experts (e.g., Chao et al., 2016; Hung et al., 2014; Lee & Johnston-Wilder, 2013), a positive learning environment and an associated self-confidence and self-efficacy of students should be a crucial element of fruitful mathematics education. The feedback from students in our FL experiment indicates that by learning and teaching mathematics in an FL, such an individualized learning environment could become possible. Similarly, our research suggests that feedback could be important for students when learning processes and learning environments are individualized. On-demand and just-in-time feedback could contribute to increasing students’ self-confidence and self-efficacy in individual learning processes in individual learning environments, and thus increasing students’ mathematics learning performance.

Creating concrete learning products could be important for students in mathematics education in general (Lee & Johnston-Wilder, 2013), and our FL experiment also illustrates that concrete learning products would be important for students. In connection with concrete learning products from students, it is evident that pride in what has been achieved is a decisive factor for students. Pride in what has been achieved goes hand in hand with students wanting feedback on their learning artifacts – feedback while working on the learning artifact and feedback at the end of a learning sequence as part of an assessment and description of a learning product.

E. Conclusion, Implications and Further Considerations
Our research aimed at exploring the elements of an FL environment could be central for students when learning mathematics with FL approaches.

Through evaluating written feedback, individual and group interviews, it can be seen that the categories (a) confidence when learning, (b) learning by working, and (c) flexibility when learning are crucial for students when learning mathematics following FL approaches. A closer look at the categories of our FL experiment allowed us to divide an FL environment into two levels: Learning activities and learning environments.

For the students, it was central in our FL experiment that they could work independently (alone or as a group) in mathematics education. Although working in mathematics education or doing mathematics was described by some students as exhausting, almost all students concluded...
that by working in mathematics education, they become more active and can discover a subject matter themselves. Thus, their learning becomes more fruitful. It could be a characteristic of mathematics education in general and according to the students’ feedback in our study in particular if mathematics is learned following flipped learning approaches that the subject matter could be discovered by students while working. However, in order to be able to work in mathematics education and thus to utilize a potentially fruitful approach to learning, several conditions should be met – namely, confidence and flexibility when learning.

Therefore, confidence and flexibility when learning could be important for students, as learning by working could require more responsibility and more meta-competencies from students. More responsibility and more meta-competencies from students could be new and additional elements of a learning process for students. For students to engage with these new and additional elements of a learning process, it might be especially important that students have a sense of confidence in the learning process. According to students’ feedback, flexibility when learning mathematics in flipped learning environments could mean that students could choose approaches or solutions to a mathematical task themselves. The possibility of choosing from several approaches or solutions while studying and working on the subject matter could be described as a typical element of mathematics. If students use these approaches and solutions independently and flexibly, as it would be typical of learning following flipped learning approaches, a learning environment’s feedback tools could be crucial. These feedback tools should allow students to check whether the approaches chosen are appropriate for solving the problem.

Learning by working in mathematics education could also lead to different students performing different activities when learning, having different needs, and requiring different meta-competencies. This variety of activities and conditions for learning by working could mean that an FL environment and process should be slightly flexible. This flexibility in FL could affect soft facts (social form or feedback) and hard facts (learning environment or materials). If mathematics education follows modern flipped learning approaches, a learning environment could be of particular significance. In general, mathematics learning environments could consist only of paper and pencil or also of modern educational technologies. For students of our flipped learning study, it was essential to switch flexibly between these partly dichotomous learning environments.

As is the case with good teachers in general (Mishra & Köhler, 2006), good mathematics teachers should have precious competencies and a vast repository of didactic and subject expertise (Goerres et al., 2015; Herreid & Schiller, 2013; Jeong et al., 2016). Our FL experiments demonstrate that teachers’ knowledge and competencies are not only important in lessons but also above all the activities and related competencies of teachers before and after lessons crucial for the success of FL mathematics education. It is only possible to learn following the principles of FL approaches in the classroom by creating a subject-specific and didactically demanding framework (preliminary phase) and adequately describing learning products and learning outcomes, as well as by post-education (post-phase).

In practice, the results of our study imply that teachers and other stakeholders in education should focus on developing and implementing student-active learning environments. Since it can be assumed that different problems or tasks will activate different students, learning environments should include a variety of tasks and approaches to explore new content. Because such an approach to teaching and learning, and this kind of learning environment could be something new for students in mathematics teaching, a transition to teaching and learning mathematics in a flipped learning environment should be done carefully. A gradual transition to teaching and learning mathematics in a flipped learning environment should contribute to students getting used to the new approach and maintaining confidence in the new way of learning.

Since we conducted our flipped mathematics learning study with young adolescent students (from 14 to 17 years old) in an upper secondary school, our research only applies to this limiting context. A further extension of our flipped mathematics learning research could be achieved by including younger students (at the beginning of secondary school), older students (at the end of secondary school), or university students into future research. Additionally, investigating teacher roles and tasks in FL environments as well as related teacher training should also be at the center of further investigations, as educated and confident teachers would be crucial in disseminating FL mathematics education. This extension of research frameworks and objectives should contribute to maintaining the quality and validity of results on crucial design elements of flipped mathematics learning environments.
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A paper presenting initial results from the first phase of our design-based research project has been presented at the CERME11 (Congress of the European Society for Research in Mathematics Education) and will be published in the conference proceedings.

F. References


