MODELING GLUCOSE AND INSULIN CONCENTRATION IN BLOOD OF HEALTHY PEOPLE AND DIABETICS

Abstract

The purpose of this study is to mathematically model the concentration of glucose and insulin in the blood in healthy people and diabetics. The dynamics of glucose and insulin concentration in the blood of healthy people and diabetics is assessed using mathematical approach. The model is derived from a glucose tolerance test mechanism that describes the behavior of physiological systems of both healthy and diabetics. The basic assumptions used in modeling the overall picture of the glucose and insulin regulation system in blood are the absence of oral glucose (food) input and the simplification of the interactions between glucose and insulin. The concentration of glucose and insulin in the blood of healthy people and diabetics is modeled using nonlinear differential equation system that contains several parameters. The model that has been established is determined the balance point and the system linearization around the balance point. The stability of the equilibrium point was then tested using the Hurwitz stability test. The results obtained show that in healthy people blood glucose concentration in the absence of glucose input, can still be maintained within a certain time according to the condition and endurance of each individual. However, this will not last long as the body continually performs metabolic processes to generate energy. While in diabetics, high blood glucose concentrations are not matched by high insulin concentrations resulting in a slower decrease in blood glucose concentration. As a result, blood glucose concentrations in diabetics remain high even in the absence of glucose input. The low concentration of insulin that is not able to maintain normal blood glucose concentration causes the time needed to lower blood glucose levels in diabetics tend to be very long compared to healthy people.

Keywords: Mathematical Model, Diabetics, Glucose, Insulin, and Concentrations

1. Introduction

High levels of glucose in the blood is closely related to the onset of symptoms of disease in humans. So the criteria, regulation, and regulation of glucose in the blood that is a vital function for every human being, it is important to know and be understood. Blood glucose concentration has an optimal level for each individual any exaggeration of this optimal concentration leads to a
severe pathological condition, which can eventually lead to death. Blood glucose levels tend to be auto-regulatory and also susceptible to various hormones that include the hormone insulin that serves to lower blood glucose concentration (Singh, 2014). Insulin is a hormone produced by a cell called beta cells of the langerhans island of the pancreas gland and serves to convert glucose into glycogen (muscle sugar) in the liver, thereby lowering glucose levels in the blood. Hipofungsi on the hormone insulin is what causes diabetes. Diabetes is a metabolic disorder in the form of loss of carbohydrate tolerance. The disease is characterized by hyperglycemia ie blood glucose levels higher than normal blood glucose levels. In the technique of autoanalysis, normal fasting glucose levels are 80 to 115 mg/dL. Diabetes is also accompanied by the emergence of complications and vascular penyakir that can cause death. The most common complications are heart attack, kidney failure, blindness, and stroke (Price dan Wilson, 2006).

Maintenance of blood glucose levels is very important, especially for maintaining nerve function. Blood glucose levels vary, depending on nutritional status. Normal human glucose levels a few hours after eating about 80 mg / dL of blood, but shortly after human eating increases to 120 mg/dL. Blood glucose levels can be determined by a glucose tolerance test. Glucose tolerance test provides a complete description of the presence of carbohydrate metabolism disorders. To determine the blood glucose levels stable or not, can be seen from the concentration of glucose and insulin in the blood. The concentration of insulin in the blood leads to the concentration of glucose in the blood. In the blood, glucose concentrations increase if the concentration of insulin decreases. Conversely, glucose concentrations decrease if the concentration of insulin increases (Fox dan Kilvert, 2010).

The effect of changes in blood glucose levels is great for human health, because glucose levels are related to the metabolic balance of important tissues in the body. To examine the problem, a mathematical approach is used. The mathematical approach in this case is mathematical modeling. The concentration of glucose and insulin in the blood is described in a mathematical model.

2. Methodology

The methodology used in this research is problem identification, literary study, formation of transfer dosage, model formulation, balance point determination, balance point classification, system linearization around equilibrium point, Jacobi matrix formation, Jacobi matrix character formation, Hurwitz stability test, numerical simulation On models, and conclusions.

3. Findings and Discussion

Mathematical model for the concentration of glucose and insulin in the blood as follows

\[
\frac{dG}{dt} = -\alpha GH - \beta G \\
\frac{dH}{dt} = -\delta H + \gamma G
\]

(1)

G : Blood glucose concentration
H : Blood insulin concentration
\(\alpha GH\) : Mass transfer of glucose to peripheral tissues that depend on insulin
\(\beta G\) : The rate of glucose transfer to the brain
\(\delta H\) : Transfer of glucose-dependent insulin mass transfer due the breakdown in plasma by enzymes
\(\gamma G\) : Extra secretion of insulin into the blood plasma that is affected by glucose due the feedback mechanism of the pangkreas.

The equilibrium point of the mathematical model of glucose and insulin concentrations in the blood without oral glucose or food input is obtained by completing the following system of equation

\[-\alpha GH - \beta G = 0 \\
-\delta H + \gamma G = 0\]

(2)
and obtained two equilibrium points \( T_1 = (G = 0, H = 0) \) dan \( T_2 = (G = -\frac{\delta \beta}{\alpha \kappa}, H = -\frac{\beta}{\alpha}) \).

This balanced state implies that the concentration of glucose and insulin in the blood does not depend on time and these two equilibrium points will be observed by the dynamics of the system as a first step. Suppose the system of differential equations (1) we write as follows:

\[
\begin{align*}
X(G,H) &= -\alpha GH - \beta G \\
Y(G,H) &= -\delta H + \kappa G \\
\end{align*}
\]

(3)

obtained by Jacobi matrix as follows

\[
J = \begin{pmatrix}
\frac{\partial X}{\partial G} & \frac{\partial X}{\partial H} \\
\frac{\partial Y}{\partial G} & \frac{\partial Y}{\partial H}
\end{pmatrix} = \begin{pmatrix}
-\alpha H - \beta & -\alpha G \\
\kappa & -\delta
\end{pmatrix}
\]

(4)

By substituting the equilibrium point \( T_1 = (G = 0, H = 0) \) in (4) linearization matrix is obtained

\[
L = \begin{pmatrix}
-\beta & 0 \\
\kappa & -\delta
\end{pmatrix}.
\]

(5)

The characteristic equation of the linearization matrix \( L \) is \(|L - rI| = 0\) or

\[
\begin{vmatrix}
-\beta - r & 0 \\
\kappa & -\delta - r
\end{vmatrix} = 0.
\]

So we get the characteristic equation \( f(r) = r^2 + (\beta + \delta)r + \beta \delta \) with \( p_o = \beta \delta > 0 \) and \( p_1 = (\beta + \delta) > 0 \) (because \( \beta > 0 \) dan \( \delta > 0 \)). Based on Hurwitz’s stability test criteria, it is concluded that the equilibrium point \( T_1 = (G = 0, H = 0) \) stable asymptotically.

For the equilibrium point \( T_2 = (G = -\frac{\delta \beta}{\alpha \kappa}, H = -\frac{\beta}{\alpha}) \), since \( \beta > 0 \) and \( \delta > 0 \), it is obtained \( G < 0 \) and \( H < 0 \). This is unlikely, because the concentration of glucose and insulin never will be negative. Therefore, stability analysis of the equilibrium point \( T_2 = (G = -\frac{\delta \beta}{\alpha \kappa}, H = -\frac{\beta}{\alpha}) \) will not be discussed.

The parameter values used for the simulation refer to Shih (1983), as follows

**Table 1.** Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Healthy People</th>
<th>Diabetics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>3.59x10^5</td>
<td>5.84x10^5</td>
</tr>
<tr>
<td>( \beta )</td>
<td>6.57x10^4</td>
<td>3.39x10^4</td>
</tr>
<tr>
<td>( \delta )</td>
<td>6.18x10^2</td>
<td>2.99x10^2</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>6.39x10^2</td>
<td>0.96x10^2</td>
</tr>
</tbody>
</table>

**Numerical Simulation of Glucose and Insulin Concentrations**

**a) On Healthy People**

By substituting the parameter values for healthy people contained in table 1. into (1), we obtain the system of nonlinear differential equations as follows:

\[
\begin{align*}
\frac{dG}{dt} &= -3.59\times10^5 GH - 6.57\times10^4 G \\
\frac{dH}{dt} &= -6.18\times10^2 H + 6.39\times10^2 G
\end{align*}
\]

(6)
By using the Maple 17, the solution curve system (6) as shown in Figure 1. This solution curve shows that the initial glucose concentration of 80 mg/100 dL, then slowly decreased due to the use of metabolic processes for the body in progress without the presence of oral glucose input from the digestive system. Blood glucose content in the absence of glucose input, can still be maintained within a certain time according to the condition and endurance of each individual body. However, this will not last long as the body continually performs metabolic processes to generate energy. The depletion of blood glucose levels due to the absence of glucose inputs results in the body losing power, even causing death. Insulin concentration in the blood will slowly run out due to its use in lowering blood glucose levels. Ending blood glucose levels cause insulin secretion in the pancreas gland to stop for a while. If there is no glucose input again, then insulin will not be secreted again.

On equilibrium point \( T_1 = (G_0, H_0)^T \), after the parameter values are inputted in table 1, we get the following linearization matrix

\[
L_1 = \begin{pmatrix}
-6.57 \times 10^{-4} & 0 \\
6.18 \times 10^{-2} & -3.59 \times 10^{-5}
\end{pmatrix}
\]

The characteristic equation of the linearization matrix \( L_1 \) is

\[
\begin{vmatrix}
-6.57 \times 10^{-4} - r & 0 \\
6.18 \times 10^{-2} & -3.59 \times 10^{-5} - r
\end{vmatrix} = 0
\]

So we get the characteristic equation,

\[
f(r) = r^2 + \left( (6.57 \times 10^{-4}) + (3.59 \times 10^{-5}) \right) r + \left( 6.57 \times 10^{-4} \right) \left( 3.59 \times 10^{-5} \right)
\]

with

\[
p_o = \left( 6.57 \times 10^{-4} \right) \left( 3.59 \times 10^{-5} \right) > 0 \quad \text{dan} \quad p_1 = \left( (6.57 \times 10^{-4}) + (3.59 \times 10^{-5}) \right) > 0
\]

Based on Hurwitz’s stability test criteria, it is concluded that the equilibrium point \( T_1 = (G = 0, H = 0) \) stable asymptotically.

b) On Diabetics

By substituting the parameter values for healthy people contained in table 1. into (1), we obtain the system of nonlinear differential equations as follows:

\[
\frac{dG}{dt} = -5.84 \times 10^{-3} GH - 3.39 \times 10^{-4} G
\]
\[
\frac{dH}{dt} = -2.99 \times 10^{-2} H + 0.96 \times 10^{-2} G
\]

(7)
By using the Maple 17, the solution curve of the system of nonlinear differential equations (7) as shown in Figure 2. This solution curve shows that the initial blood glucose concentration of 130 mg/100 dL in the first minute and high was not matched by the insulin high concentration causes a slow decrease in blood glucose concentration. As a result, the blood glucose content in diabetics remains high even in the absence of glucose input. The time taken to lower blood glucose levels in diabetics tends to be very long compared to normal people due to the low concentration of insulin that is not able to maintain normal blood glucose concentration. If diabetics impose excessive carbohydrate diet, it can cause hypoglycemia because the concentration of glucose and insulin in the blood tends to run out due to the absence of glucose and insulin input in the long term.

On equilibrium point  \( T_1 = (G=0, H=0) \), after the parameter values are inputted in table 1, we get the following linearization matrix

\[
L_2 = \begin{pmatrix}
-3.39 \times 10^{-4} & 0 \\
2.99 \times 10^{-2} & -5.84 \times 10^{-5}
\end{pmatrix}
\]

(15)

The characteristic equation of the linearization matrix \( L_2 \) is

\[
f(r) = r^2 + \left( (3.39 \times 10^{-4} r) + (5.84 \times 10^{-5} r) \right) r + \left( 3.39 \times 10^{-4} \right) \left( 5.84 \times 10^{-5} \right) = 0
\]

So we get the characteristic equation,

\[
p_o = (3.39 \times 10^{-4}) (5.84 \times 10^{-5}) > 0 \text{ and } p_1 = \left( 3.39 \times 10^{-4} \right) + \left( 5.84 \times 10^{-5} \right) > 0.
\]

Based on Hurwitz's stability test criteria, it is concluded that the equilibrium point  \( T_1 = (G=0, H=0) \) stable asymptotically.

Research on the model of glucose and insulin concentrations in the blood of healthy people and diabetics has not paid attention to oral glucose (food) inputs so as to derive a more accurate model describing a more tangible phenomenon preferably taking into account the oral glucose input factor (food).

4. Conclusion

The dynamics of glucose and insulin concentrations in the blood of healthy people and diabetics are different. In healthy people, high blood glucose concentrations are offset by high insulin concentrations as well. So that blood glucose levels in healthy people can be maintained properly. While in diabetics, high blood glucose concentration is not matched by high concentrations of insulin as well. So the blood glucose level of diabetics is always high beyond normal levels.
5. References


