PREVENTIVE MAINTENANCE OPTIMIZATION FOR PRODUCT WARRANTY TWO-DIMENSIONAL BY ONE DIMENSIONAL APPROACH

ABSTRACT

This paper discusses how to determine the optimal time interval for Preventive Maintenance in the areas warranty (warranty Baseline and Extended Warranty) which minimizes the cost of the warranty. The technique is done is one-dimensional approach by dividing the area into two regions guarantee users that users who use rate of below and above the baseline rate of usage. Because each of the areas taken, the average usage rate that is expected to describe the rate of use of each region. Furthermore, the optimal period for PM searched using an analytical approach. The results of the model can be used to determine the extended warranty is right for producers. Numerical examples are provided to illustrate the results obtained.

KEYWORDS: Optimization, Preventive Maintenance, one-dimensional approach.

1. Introduction

The concept of optimization plays a very important in manufacturing both for producers and consumers or other parties such as OEM. Optimization concepts discussed in this paper is based on the viewpoint of the manufacturer. Manufacturers take action Preventive Maintenance in the area and offer a warranty on a consumer Warranty Extended.

A care measure for the system is divided into two broad categories, Preventive Maintenance (PM) and Corrective Maintenance (CM) (Huang etc, 2014). The difference between the two is the PM is an action in a structured or scheduled maintenance while the CM is not. CM is only done when there is damage to a system.

Warranty cost is an accumulation of charge Corrective Maintenance. To lower warranty cost, the amount of damages should be minimized. The usual approach by the manufacturer is applying measures in the area of preventive maintenance warranty. Of course, this approach is done if the resulting reduction in warranty cost exceeds the cost of the PM. The magnitude of the effect of PM on the decline in warranty cost depends on the election schedule PM appropriate. Therefore, it needs optimal maintenance policies to control the damage during the warranty period.

Literature that discusses optimization Preventative maintenance for product warranty of one dimension can be seen in (Pascual etc, 2006), (Pascual etc, 2012) and (Huang etc, 2014)
while for product warranty of two dimensions can be found in (Wang et al., 2015), (Husniah et al., 2013) and (Jiang et al., 2008).

(Pascual and Ortega, 2016) develop models to aid decision-making in action PM by determining the optimal time for an overhaul by minimizing repair costs per unit of time. Furthermore, (Pascual and Godoy, 2012) to re-implement the strategy of preventive maintenance as (Pascual, 2016) to maximize the total profit expected by manufacturers and contractors who usually try to optimize profits separately. In particular, this method find optimal preventive maintenance interval between contractors, manufacturers and service chain.

For product warranty of two-dimensional, (Huang et al., 2014) examines the cost analysis by applying the periodic PM and performed by Bivariate Weibull approach. (Huang et al., 2014) determines the optimal time warranty to maximize profits from the manufacturer. Meanwhile, the optimal strategy for the regional periodic PM Base Warranty and Extended Warranty assessed (Wang et al., 2015). However, this strategy examines the optimization PM on maintenance varying degrees. (Wang et al., 2015) it does not provide an explanation at the level (maintenance degrees) where the time between the PM achieve the most optimal. Furthermore, (Wang et al., 2015) also did not provide clarification regarding the policy that is most optimal Extended Warranty that must be applied by the manufacturer.

In this paper, we develop a model in (Jiang et al., 2008) of the one-dimensional to two-dimensional. The model proposed in this paper is a model that can determine the optimum PM intervals for product warranty of two-dimensional and the method used is the method of analytic approach. Furthermore, this model can provide results on which level (maintenance degrees) optimal PM interval is reached. Further we can also provide Extended Warranty optimal region to maximize producer profits.

An outline of this paper is as follows. In the second part described about the model formulation. Section 3 describes the analysis models. In section 4 Numerical examples are given to illustrate the optimal solution obtained from the model and the latter was given a conclusion with a brief discussion of a topic for further research.

2. Model Formulation

2.1 Notation

The notation used is as follows:

$T$ : Random variables of age (non-negative)
$R$ : Random variables of usage rate
$g(r)$ : Function chances of $R$
$f(t)$ : Function chances of $T$
$[0, W_i) \times [0, U_i) : Base Warranty (BW)$ region
$[0, W_i) \times [0, U_i) : Extended Warranty (EW)$ region
$N_i(W, U)$ : The amount of damage in each region. $i = 1$ stating the rate of use of light and $i = 2$ stated rate of heavy use.
$\tau_i$ : Time between $PM$ ($i = 1,2$)
$k_i$ : The amount of damage in the area $BW$
$l_i$ : The amount of damage in the area $EW$
$C_m$ : Minimal cost repair
$C_0$ : Standard cost $PM$
$C_r$ : The cost of quality improvement
$\gamma_0$ : Baseline of the rate of use in $BW$
$\gamma_1$ : Baseline of the rate of use in $EW$
$\lambda_0(t)$ : The function of the intensity of the initial damage
$\lambda_j(t)$ : The function of the intensity after the change to $j$.
$\delta_j$ : The reduction of the intensity function after $PM$
2.2 Failure Model of Vehicles

In the one-dimensional approach, the usage is considered as a function of age \( t \). It is also assumed that the relationship between the two is linear and determines the coefficient of non-negative direction. Relationship can be written as: \( Y(t) = rt \) [4]. Suppose \( \tilde{N}(t|r) \) is a non-negative random variable that specifies the number of damage to the hose \([0, t]\) with condition \( R = r \), thus \( \tilde{N}(t|r) \) follows the Poisson process with intensity function Non-Homogeneous:

\[
\lambda(t|r) = \theta_0 + \theta_1r + \theta_2t + \theta_3rt
\]  

(1)

If \( L \) declare the expiration of the warranty period is conditional usage rate \( R = r \) the meaning of the image (1) it can be seen that:

\[
L = \begin{cases} 
W_0 & \text{for } r \leq \gamma \\
\frac{U_0}{r} & \text{for } r > \gamma 
\end{cases}
\]  

(2)

By using properties of conditional expectation, the expectation of the amount of damage to the system in the area of warranty \( \Omega_w \) is:

a. The case of utilization rates \( r \leq \gamma_0 \)

\[
E[N_i(W, U)] = E\left[ E\left[ N(W, U|r) \right] \right] = \int_0^W \left( \int_0^r \lambda(t|r) dt \right) g(r) dr
\]  

(3)

b. The case of utilization rates \( r > \gamma_0 \)

\[
E[N_i(W, U)] = E\left[ E\left[ N(W, U|r) \right] \right] = \int_{\gamma_0}^\infty \left( \int_0^r \lambda(t|r) dt \right) g(r) dr
\]  

(4)

Next we will create the formula above is simplified. Suppose \( \tilde{r_i} \), \( i = 1, 2 \) assumed to represent each region then for \( R = \tilde{r_i} \) intensity function turns into

\[
\lambda(t) = \lambda(t|\tilde{r_i}) = \left( \theta_0 + \theta_1\tilde{r_i} \right) + \left( \theta_2 + \theta_3\tilde{r_i} \right) t ; \ i = 1, 2
\]  

(5)

wherein

\[
\tilde{r_i} = \frac{1}{G(r_i) - G(r_{i-1})} \int_{r_{i-1}}^{r_i} r g(r) dr, \ i = 1, 2
\]  

(6)

\( r_0 = R_L, \ r_i = R_L + i \frac{R_U - R_L}{2}, \ R_L \) and \( R_U \) determining the lower and upper limits of the rate of use \( R \).
Because \( r \) is a constant, then the above functions can be rewritten as \( \lambda(t) = \alpha_0 + \alpha t \), means that the function of intensity is directly proportional to time and other factors, including user error. In this paper we assume no damage caused by misapplication therefore \( \alpha_0 = 0 \) so the intensity function is

\[
\lambda(t) = \alpha t
\]  (7)

Therefore the expectations of the extensive damage in the area guarantees are as follows:

a. Case (1): The area with the rate of use \( r \leq r_0 \)

\[
E[N_1(W_0,U_0)] = \int_0^W \lambda(t)dt \int_0^r g(r)dr = G_{\alpha_1} \int_0^W \lambda(t)dt
\]  (8)

b. Case (2): The area with the rate of use \( r > r_0 \)

\[
E[N_2(W_0,U_0)] = \int_0^W \lambda(t)dt \int_r^\infty g(r)dr = G_{\alpha_2} \int_0^W \lambda(t)dt
\]  (9)

2.3 Failure model of Vehicles

![Figure 2. Intensity function](image)

Each PM to \( j \) will reduce the intensity function so that the function of the intensity after the PM to \( j \) is

\[
\lambda_j(t) = \lambda_i(t) - \sum_{i=1}^j \delta_i
\]  (10)

\[
\int_0^L \lambda_j(t)dt = \sum_{j=1}^{k+1} \int_{t_{j-1}}^{t_j} \lambda_{j-1}(t)dt
\]  (11)

By outlining the right hand side of the above equation is obtained

\[
\int_0^L \lambda_j(t)dt = \lambda(L) - \sum_{j=1}^{k} (L-t_j) \delta_j
\]  (12)

3. Model Analysis

In a certain time interval, do the PM as much as \( k \) times so as to determine the optimal interval between PM, the interval is divided into \( (k + 1) \) parts. By using the concept of derivative, and for a fixed \( k \), obtained the desired.

3.1 Expectation of Cost Guarantee

Furthermore, expectations of the extensive damage to each region after the PM as much as \( k \) times is
Suppose expected warranty costs after PM to \( k \) is
\[
E[C_i] = k_i C_{pm} + C_w E_m[N_i(W,U)], \quad i = 1,2
\]
If the cost of PM after repairs are given by \( k_i C_o + C_v \sum_{j=1}^{k} \delta_j \)
meaning expectations warranty costs after PM to \( k \) is
\[
E[C_i] = G_o C_o \Lambda(L) - \left[ \sum_{j=1}^{k} (C_o G_0 (L - j T_o) - C_v) \delta_j - k_i C_o \right]
\]
Our goal is to minimize the equation above. This equation reaches a minimum if the second syllable diruas right side reaches its maximum. In order for the second term on the right-hand side reaches its maximum then \( \delta_j = \delta_{j,\max} \). Note that \( 0 \leq \delta_j \leq \lambda(t_j) - \sum_{i=1}^{j-1} \delta_i \). In order to \( \delta_j = \delta_{j,\max} \) then it must be \( \delta_j = \lambda(t_j) - \sum_{i=1}^{j-1} \delta_i \). From here we get an objective function, namely
\[
J(k, \tau_i) = \left[ \sum_{j=1}^{k} (C_o G_o (L - j T_o) - C_v) \left( \lambda(j T_o) - \lambda((j-1) T_o) \right) \right] - k_i C_o
\]
Through the concept of derivative, for a fixed \( k \), obtained the critical point that satisfies the above equation, namely
\[
\tau_i^* = \frac{L - C}{G_0 C_w} \left( k_i + 1 \right)
\]
To check the nature of the critical point is used to test the second derivative and obtained
\[
\frac{d^2}{d \tau_i^2} J(k, \tau_i) = -G_0 C_o \delta k_i (k_i + 1) < 0
\]
From here obtained the second derivative is negative for each \( \tau_i \), means that \( J(k, \tau_i) \) reaches its maximum at the point \( \tau_i^* \).

### 3.2 Expansion of Regional Warranty

Manufacturers agreed to expand the area of warranty if expectations for additional warranty costs do not exceed expectations basic warranty costs [2] so
\[
E_{pm}[C_{aw}] + E_{pm}[C_{ew}] \leq E[C_{aw}]
\]
From here we get
\[
E_{pm}[C_{ew}] \leq J(k, \tau_i)
\]
These conditions must be met so that the producers do not lose.

If manufacturers want to offer Extended Warranty to consumers then there are three things to note: (1) The case of \( \gamma_1 > \gamma \); (2) Kasus \( \gamma_1 < \gamma \); (3) Kasus \( \gamma_1 = \gamma_0 \).

1. Case \( \gamma_1 > \gamma_0 \)
   This case is also divided into three cases are as follows:
   1.1 Case \( r \leq \gamma_0 \)
   In this case, the warranty period for the consumer ends on \( W_0 + W_i \). If the PM BW done as much as \( k \) times the intensity function after PM to \( k \) is \( \lambda_k(t) \). To determine the interval PM in the period
of EW, it is the same way by determining the interval PM during BW. It should be noted that the function of the initial intensity in the future BW is \( \lambda_0(t) \) while the function of the initial intensity during EW is \( \lambda_1(t) \).

1.2 Case \( r_0 < r \leq r_1 \)

In this case, the warranty period for the consumer ends on \( U_0/r + W_1 \).

1.3 Case \( r > r_1 \)

For all cases, the method of calculation to determine the optimal time between the PM is sama. Yang noteworthy is the expiration of the warranty of a consumer. In this case, the warranty period for the consumer ends on \( U_0/r + U_1/r \).

![Figure 4. Warranty area for \( r \leq r_0 \)](image)

![Figure 5. Warranty area for \( r > r_1 \)](image)

2. Case \( r_1 < r_0 \)

This case is also divided into three as in cases (1). How to determine the optimum PM intervals within the warranty period is also similar to the way in the first case.

3. Case \( r_1 = r_0 \)

The case is divided into two:

3.1 Case \( r < r_1 = r_0 \)

3.2 Case \( r > r_1 = r_0 \)

model of described in the previous chapter can be directly used to determine how much time between PM optimal in all areas of the warranty (BW and EW). If the baseline of the rate of use in BW and EW are different, there is the possibility of time between the PM of the two areas is also different because this method separately calculate how much time between PM optimal to minimize warranty cost in the region BW and EW. However, if the baseline rate of consumption in the same area, then the time will be the same between her PM.

4. Example of Numeric

In this section performed numerical simulations to apply models that have been created. In this simulation we use cases (1.1). Assumed product warranty period is 2 years or \( 2\times10^4 \text{ km} \) and EW is 2 years or \( 4\times10^4 \text{ km} \). The rate of consumption is assumed to follow a uniform distribution \( U \cup (0,6) \), random variables of age assumed by Weibull distribution parameters \( \alpha = 2 \) and \( \beta = 1.5 \) and the use of random variables derived from the relationship between random variables, namely age and usage rate as a function of age.
Data generation for random variables of age, the rate of consumption and the use of above separated by the magnitude of the rate of usage. Age data for the case (1) by the symbol $1X$, and age data for the case (2) given the symbol $2X$. This data each have a distribution function $f_i$ and $f_2$ with:

$$f_i(x) = (\theta_i x)^{i-1}e^{-\theta_i x}, \quad i = 1, 2$$

(22)

This function is obtained from the correlation function of the intensity, pace hazard and function chances of a random variable. [4]. With a Maximum Likelihood method is obtained $\theta_1 = 0.4735$ and $\theta_2 = 0.49171$.

Suppose the cost of corrective and preventive maintenance in succession is $C_m = $100, $C_c = $100, $C_r = $50. As an example we take the consumer with a low usage rate. Of the costs obtained results of the calculation as follows.

The optimal value for $(k_i, \tau_i)$ The maximum that can function in equation (1.4) is $k^*_i = 3$ and $\tau^*_i = 5.25$ bulan months with maximum values of $1,005.14$. Expectation of warranty costs for the case (1) is approximately $2,272.90$ so that the function of the equation (1.4) satisfies the minimum amount of $1,267.76$ for $k^*_i = 3$, $T^*_i = 5.25$ months and $\delta^* = 2.486$.

Some values for $k_i$ and $\tau_i$ can be seen in Table 1, and plot the drawing can be seen in Figure 6. For EW area, the result can be seen in Table 2 and Figure 7.

Expectation of minimal warranty cost is obtained by performing PM three times in BW and five times in EW. Interval PM for areas BW is 5.25 months and 3.75 months with EW is $\delta^* = 2.486$ and $\delta^* = 1.776$ in succession.

However, if the manufacturer wants RW most optimal limit, then the condition (18) must be met. This condition is met if the age limit to EW is 15 months and the use of $2.5 \times 10^4 km$. The results can be seen in Table 3 and Figure 8.

### Table 1. Interval of PM optimal in BW

<table>
<thead>
<tr>
<th>$k_i$</th>
<th>$\tau_i$</th>
<th>$\delta$</th>
<th>$E[C_{bw}]$</th>
<th>$J(k_i, \tau_i)$</th>
<th>$E_{pm}[C_{bw}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.5</td>
<td>4.972</td>
<td>$2,272.9$</td>
<td>$770.09$</td>
<td>$1,502.80$</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3.315</td>
<td>$2,272.9$</td>
<td>$960.12$</td>
<td>$1,312.77$</td>
</tr>
<tr>
<td>3</td>
<td>5.25</td>
<td>2.486</td>
<td>$2,272.9$</td>
<td>$1,005.14$</td>
<td>$1,267.76$</td>
</tr>
<tr>
<td>4</td>
<td>4.2</td>
<td>1.989</td>
<td>$2,272.9$</td>
<td>$992.15$</td>
<td>$1,280.75$</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>1.657</td>
<td>$2,272.9$</td>
<td>$950.16$</td>
<td>$1,322.74$</td>
</tr>
</tbody>
</table>

### Table 2. Interval of PM optimal in EW

<table>
<thead>
<tr>
<th>$l_i$</th>
<th>$\tau_i$</th>
<th>$\delta$</th>
<th>$E[C_{ew}]$</th>
<th>$J(l_i, \tau_i)$</th>
<th>$E_{pm}[C_{ew}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.25</td>
<td>5.327</td>
<td>$4,545.8$</td>
<td>$1,897.66$</td>
<td>$2,648.13$</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>2.131</td>
<td>$4,545.8$</td>
<td>$2,796.26$</td>
<td>$1,749.53$</td>
</tr>
<tr>
<td>5</td>
<td>3.75</td>
<td>1.776</td>
<td>$4,545.8$</td>
<td>$2,829.44$</td>
<td>$1,716.35$</td>
</tr>
<tr>
<td>6</td>
<td>3.214</td>
<td>1.522</td>
<td>$4,545.8$</td>
<td>$2,824.56$</td>
<td>$1,721.23$</td>
</tr>
<tr>
<td>7</td>
<td>2.813</td>
<td>1.332</td>
<td>$4,545.8$</td>
<td>$2,795.91$</td>
<td>$1,749.88$</td>
</tr>
</tbody>
</table>
Table 3. Optimal area of $EW$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\tau_i$</th>
<th>$\delta$</th>
<th>$E[C_{EW}]$</th>
<th>$J(l_i,\tau_i)$</th>
<th>$E_{pm}[C_{EW}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.75</td>
<td>3.196</td>
<td>$1,775.7$</td>
<td>$619.16$</td>
<td>$1,156.54$</td>
</tr>
<tr>
<td>2</td>
<td>4.500</td>
<td>2.131</td>
<td>$1,775.7$</td>
<td>$758.88$</td>
<td>$1,016.82$</td>
</tr>
<tr>
<td>3</td>
<td>3.375</td>
<td>1.598</td>
<td>$1,775.7$</td>
<td>$778.74$</td>
<td>$996.96$</td>
</tr>
<tr>
<td>4</td>
<td>2.700</td>
<td>1.279</td>
<td>$1,775.7$</td>
<td>$750.65$</td>
<td>$1,025.05$</td>
</tr>
<tr>
<td>5</td>
<td>2.250</td>
<td>1.065</td>
<td>$1,775.7$</td>
<td>$698.60$</td>
<td>$1,077.10$</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper has been learned about how to determine the optimal time for preventive maintenance on a product warranty of two dimensions by using a one-dimensional approach. Optimal time obtained analytically. Beside the time delta between the PM also obtained optimal for each action PM. The results can be used directly to calculate how much the PM done in the area warranty (Basic and Extended Warranty). Furthermore, this method can be used to determine the optimal extension of the area for the manufacturer's warranty.

References


