THE APPLICATION OF DUAL RECIPROCITY BOUNDARY ELEMENT METHOD FOR STEADY INFILTRATION PROBLEMS IN THE PERIODIC CHANNEL WITH IMPERMEABLE LAYER

Abstract

In this paper, we will discuss Dual Reciprocity Boundary Element Method (DRBEM) for steady infiltration problems of various types of channels that vary in a periodic channel on homogeneous soil. Infiltration homogeneous stationary on the ground constructed Richards Equation. Richards's equation is then transformed into a modified Helmholtz equation. Furthermore, with DRBEM, the numerical solution of the modified Helmholtz equation is obtained. By using numerical solution obtained, can be calculated numerical value of suction potential.

Keywords: Infiltration, periodic channel, boundary element method, dual reciprocity

1. Introduction

Water is an indispensable element by plants and all things related to agricultural activity. With the availability of sufficient water can be a factor supporting increased agricultural output. Yet it cannot be denied, not every area or farmlands have sufficient water availability. It is caused by low rainfall so it will be very difficult to meet the water needs of plants. In connection with the foregoing, the irrigation system is necessary to overcome the problem of agricultural land with less water intensity. Their irrigation system is expected to maintain level high enough moisture in the soil so that plant roots can extract water from the soil efficiently.

Along with the development of science, much research has been done with regard to infiltration of irrigation channels. One of them is the research on Steady infiltration from a ditch. However, these problems are only solved analytically. For a more realistic case, a numerical method is required. In addition, although the analytical method can only be used to a certain extent but it is useful to develop alternative numerical approach. One approach worth considering is the Dual Reciprocity Boundary Element Method is known for flexibility in resolving the boundary conditions in a lot of trouble. Therefore, the authors are interested to know the application of the Dual Reciprocity Boundary Element Method (DRBEM) for stationary infiltration problems in different types of channels periodically on the type of soil impermeable homogeneous.
2. Problem Formulation

The following will be given formulation infiltration problems from different channels on different irrigation homogeneous soil. The irrigation channels that will be compared with each other (see Figure 1) is:

a. Rectangular channel with an impermeable layer.

b. Trapezoidal channel with an impermeable layer.

Next will be determined each domain of irrigation channels above. In determining the domain of irrigation channels, should be given specific assumptions that:

a) The length of each cross-section of irrigation channels are the same, namely 2L

b) The distance between channels is the same, namely 2D, or the distance between the midpoint of the channel is the same that 2(L + D)

c) Channels are very long

d) Channel is always filled with water

e) Infiltration on the surface of irrigation channels is constant that is v0

f) No infiltration in the ground outside the irrigation channel

g) No infiltration on the surface of permeable channels.

![Figure 1. Irrigation canals periodic](image)

By symmetry, can be defined as a semi-infinite domain that 0 ≤ X ≤ L + D and Z ≥ 0 stated by R with limitation C, and no flux at X = 0 and X = L+D.

3. Fundamental Equation

For stationary infiltration problems, the mathematical models often used are:

\[
\frac{\partial}{\partial X} \left( K \frac{\partial \psi}{\partial X} \right) + \frac{\partial}{\partial Z} \left( K \frac{\partial \psi}{\partial Z} \right) = \frac{\partial K}{\partial Z}
\]  

(1)

K is the hydraulic conductivity of unsaturated soil and Z > 0 leads down vertically. Equation (1) is nonlinear partial differential equations representing the movement of water in unsaturated soil in two dimensions often called Richards equation.

According to [1] in the paper explained the potential flux matrix Θ associated with hydraulic conductivity by Equation

\[
\Theta = \int_{-\infty}^{\psi} K(q) dq
\]  

(2)

The relationship between K hydraulic conductivity of unsaturated soil (unsaturated soil) and K0 hydraulic conductivity of saturated soil (saturated soil) is defined as

\[
K = K_0 e^{\alpha \psi}, \quad \alpha > 0,
\]  

(3)

\[\leftrightarrow \alpha \Theta = K, \]  

(4)

with ψ is suction potential and α is an empirical constants.

Using Equation (2), (3) and (4) obtained by the linear form of Equation steady infiltration namely

\[
\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Z^2} = \alpha \frac{\partial \Theta}{\partial Z}
\]  

(5)

Flux component to the horizontal and vertical directions is

\[
U = -\frac{\partial \Theta}{\partial X}, \quad \text{and} \quad V = \alpha \Theta - \frac{\partial \Theta}{\partial Z}
\]  

(6)

Defined dimensionless variables as follows
\[ x = \frac{\alpha}{2} X; \quad z = \frac{\alpha}{2} Z; \quad \Phi = \frac{\pi \Theta}{v_i L}; \quad u = \frac{2\pi}{v_i \alpha L} U; \quad v = \frac{2\pi}{v_i \alpha L} V; \quad f = \frac{2\pi}{v_i \alpha L} F, \]  

and substitute \( \Phi = \phi e^{iz} \) into equation (5) obtained an equation that is shaped Helmholtz Equation

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \phi. \]  

Equation (8) is a modified Helmholtz equation.

Dimensionless flux obtained

\[ f = -e^{iz} \left[ \frac{\partial \phi}{\partial n} - \phi n_{z2} \right] \Leftrightarrow \frac{\partial \phi}{\partial n} = \phi n_{z2} - fe^{iz}. \]  

Flux on the surface of the channel is \( v_0 \) and large dimensionless flux is \( 2\pi / \alpha L \). So that the boundary condition on the surface of permeable channels is

\[ \frac{\partial \phi}{\partial n} = \frac{2\pi}{\alpha L} e^{iz} + \phi n_{z2}. \]  

The absence of flux permeable channels, ground level outside the channel and along the \( X = 0 \) and \( X = L + D \), resulting in

\[ \frac{\partial \phi}{\partial n} = \phi n_{z2}, \quad \text{on the surface of impermeable channel} \]  

\[ \frac{\partial \phi}{\partial n} = \phi, \quad \text{on the surface outside of the channel} \]  

\[ \frac{\partial \phi}{\partial n} = 0, \quad x = 0 \quad \text{and} \quad z \geq 0 \]  

\[ \frac{\partial \phi}{\partial n} = 0, \quad x = \frac{\alpha}{2} (L + D) \quad \text{and} \quad z \geq 0 \]  

Given assumption [3] namely

\[ \frac{\partial \Theta}{\partial X} \rightarrow 0, \quad \text{and} \quad \frac{\partial \Theta}{\partial Z} \rightarrow 0 \quad \text{for} \quad Z \rightarrow \infty \]  

So that

\[ f = 2\phi e^{iz}. \]  

Using equation (10) gained

\[ \frac{\partial \phi}{\partial n} = 0, \quad x = \frac{\alpha}{2} (L + D) \quad \text{and} \quad z = \infty \]  

Dual Reciprocity Boundary Element Method (DRBEM) used to find numerical solutions Equation (8) with the boundary condition given in Equation (10) - (16). At a later stage, look for the numerical solution with the required DRBEM an integral equation called boundary integral equation is

\[ \lambda(\xi, \eta) \phi(\xi, \eta) = \int_C \left[ \phi(x, z) \frac{\partial \Phi(x, z; \xi, \eta)}{\partial n} - \Phi(x, z; \xi, \eta) \frac{\partial \phi(x, z)}{\partial n} \right] ds(x, z) \]

\[ + \int_R \Phi(x, z; \xi, \eta) \phi(x, z) dxdz, \]  

with

\[ \Phi(x, z; \xi, \eta) = \frac{1}{2\pi} \ln \sqrt{\left[(x - \xi)^2 + (z - \eta)^2\right]} \]  

is the fundamental solution of the dual-dimensional Laplace equation

\[ \lambda(\xi, \eta) = \begin{cases} 0, & \text{if} \ (\xi, \eta) \notin R \cup C \\ \frac{1}{2}, & \text{if} \ (\xi, \eta) \text{ laid on the smooth curve } C \\ 1, & \text{if} \ (\xi, \eta) \in R \end{cases} \]
The integral equation (17) can be solved numerically by discretization on domain boundaries using segments of lines connected to each other by their edges lie on the curve C and discretization domain with interior collocation point. Suppose the line segments $C^{(k)}$, $k = 1, 2, ..., N$ so that the number of line segments as $N$ segment. For each $C^{(k)}$, $k = 1, 2, ..., N$, selected midpoint as collocation points that was written with $(a^{(1)}, b^{(1)}), (a^{(2)}, b^{(2)}), ..., (a^{(N)}, b^{(N)})$ and points collocation interior of $L$ point that can be written as $(a^{(N+1)}, b^{(N+1)})$ or $(a^{(n)}, b^{(n)})$ for $n=1, 2, ..., N+L$, then approximation can be written as

$$\lambda(x^{(n)}, y^{(n)}) \phi^{(n)} = \sum_{j=1}^{n} \mu^{(j)} + \sum_{k=1}^{N} \left( F_1^{(k)}(x^{(n)}, y^{(n)}) \right) - p^{(k)} F_1^{(k)}(x^{(n)}, y^{(n)})$$

(20)

with $\phi^{(n)} = (x^{(n)}, y^{(n)}) (n=1, 2, ..., N+L)$ and

$$\mu^{(n)} = \sum_{j=1}^{n} \omega^{(m)} \Psi(x^{(n)}, y^{(n)}, x^{(m)}, y^{(m)})$$

(21)

$$F_1^{(k)}(\xi, \eta) = \int_{C^{(k)}} \Phi(x, z; \xi, \eta) ds(x, z)$$

(22)

$$F_2^{(k)}(\xi, \eta) = \int_{C^{(k)}} \left[ \partial \Phi(x, z; \xi, \eta) \right] ds(x, z)$$

(23)

and $\Psi(\xi, \eta; a, b) = \lambda(\xi, \eta) \chi(x, y; a, b) \left[ \sum_{j=1}^{n} \left[ n_{ij} \partial \chi(x, y; a, b) + n_{ij} \partial \chi(x, y; a, b) \right] \right]$

$$F_1^{(k)}(\xi, \eta) - \sum_{j=1}^{N} \chi(x^{(k)}, y^{(k)}; a, b) F_2^{(k)}(\xi, \eta).$$

(24)

$$\omega^{(m)} = \left[ \rho(x^{(j)}, y^{(j)}; x^{(m)}, y^{(m)}) \right]^{-1}$$

(25)

with

$$\rho(x, z; a^{(m)}, b^{(m)}) = 1 + r^2 \left( x, z; a^{(m)}, b^{(m)} \right) + r^3 \left( x, z; a^{(m)}, b^{(m)} \right),$$

$$\chi(x, z; a^{(m)}, b^{(m)}) = \frac{1}{4} r^2 \left( x, z; a^{(m)}, b^{(m)} \right) + \frac{1}{16} r^4 \left( x, z; a^{(m)}, b^{(m)} \right) + \frac{1}{25} r^5 \left( x, z; a^{(m)}, b^{(m)} \right),$$

$$r(x, z; a^{(m)}, b^{(m)}) = \sqrt{(x-a^{(m)})^2 + (z-b^{(m)})^2}.$$ 

DRBEM used to obtain the numerical solution ($\phi$), and use the following equation to obtain suction potential ($\psi$).

$$\psi = \frac{1}{\alpha} \ln \left( \frac{\alpha v_L \Delta \rho e^2}{\pi K_0} \right).$$

(26)

To obtain a water content (\theta) used Equation

$$\frac{\theta_t - \theta_r}{\theta_t - \Theta_r} = \left[ \frac{1}{1+(\alpha \psi)^n} \right]^m$$

(27)

with $\theta_r$ and $\Theta_r$ is the residual water content and saturated, $\alpha$, $n$ and $m = 1 - (1/n)$ is a parameter that depends on the type of soil.

3. Findings and Discussion

DRBEM applied to the problem of steady infiltration on multiple channels periodically varying in homogeneous soil and selected types of soil Pima clay loam (PCL) with the value of $\alpha$ and $K_0$ respectively 0.014 cm$^{-1}$ and 9.9 cm/day [5], [6], $\theta_r = 0.095$ cm$^{-1}$, $\theta_s = 0.41$ cm$^{-1}$ and $n = 1.31$ cm$^{-1}$ [7], and the value of $v_0$ is 75% of the value of $K_0$ [2]. Furthermore, the size of each non-flat impermeable channel can be selected so that the surface of the same channel length is $L = 50$ cm and $D = 50$ cm. Using the $L$ value, the non-flat channel width is 200/\mu cm with a depth of 50 cm for rectangular channels with impermeable and 49.5625 cm layer on trapezoidal channel with

{$$\psi = \frac{1}{\alpha} \ln \left( \frac{\alpha v_L \Delta \rho e^2}{\pi K_0} \right).$$$

To obtain a water content (\theta) used Equation

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with $\theta_r$ and $\Theta_r$ is the residual water content and saturated, $\alpha$, $n$ and $m = 1 - (1/n)$ is a parameter that depends on the type of soil.
an impermeable layer. Furthermore, DRBEM is implemented in MATLAB language programming.

To obtain a numerical solution (θ), the domain of the problem to be limited by a simple closed curve. In other words, the value of z in the domain is in the range of $0 < z < c$ with $c$ positive real numbers. Based on several studies that have been there before, $c = 4$ is good enough condition to be applied at the domain boundaries without any significant effect on the value of θ in the domain. So that the domain boundary at the deepest part is $z = 4$.

In DRBEM, the number of line segments (N) and an interior point (M) affect the accuracy of numerical solutions generated. In this case, given the number of line segments (N) with the midpoint as collocation points and interior points (M) of each channel that is:

1. Rectangular channel with a layer of impermeable given $N = 208$ and $M = 609$
2. Trapezoidal channel with a layer of impermeable given $N = 209$ and $M = 609$.

Furthermore, variations in the value of ψ and θ with depth (Z) at different distances from the center line (X) for the geometry can be seen in Figure 2-6 below.

![Figure 2. Suction potential and water content from various channels at X = 15](image)

From Figure 2 (a) is at $X = 15$ which is located beneath the channel, the value of ψ for all types of channels increased to Z. This shows that the water content (water content) in the bottom of the channel in the shallow part is smaller than in the deepest part of the soil so that water seepage is getting into even greater. This is supported by the declining value of θ (see Figure 2 (b)) which indicates that the water content further down the greater. This is because the position of $X = 15$ is located in the bottom of the channel impermeable so that based on the initial assumption was no infiltration.

![Figure 3. Suction potential and water content from various channels at X = 30](image)

As to the explanation of Figure 2 is $X = 30$ indicates a decrease in the value of ψ and θ to Z as it is below the permeable channels. This shows that the water content (water content) in the bottom of the channel in the shallow part is greater than in the deepest part of the ground so water seepage is getting into increasingly smaller. This is supported by the declining value of θ (see Figure 3 (b)) which indicates that the water content further down the smaller.
Figure 4. Suction potential and water content from various channels at X = 50

In a channel with an impermeable layer (Figure 4 (a)), the value $\psi$ ride at a certain depth and down at certain depths also heading to its converge. It shows that in the shallow part to a certain depth below the surface of the soil moisture content is lower than in the deeper part. This is illustrated in Figure 4 (b) the value of $\theta$ up and down to get to its converge.

Figure 5. Suction potential and water content from various channel at X = 65

At X = 65 and X = 85, the value of $\psi$ increased to Z (Figure 5 (a) and 6 (a)). This shows that the water content (water content) on the ground level is the lowest compared to all parts of the land within the domain. it is also shown in Figure 5 (b) and 6 (b) the value of $\theta$ increases. This is because the initial assumption that the ground level outside the channel there is no flow of water into the soil.

Below we will give the distribution of values $\psi$ of non-flat channel with a layer of impermeable geometrically presented in Figures 7 and 8 below.

Figure 6. Suction potential and water content from various channel at X = 85

Figure 7. value distribution of $\Psi$ in rectangular channel with an impermeable layer
From Figure 7 and 8 above, it is clear distribution of values $\psi$. In the shallow subsurface channel has a value of $\psi$ greatest among another section and continue downhill towards the point to its converge. This shows that the water content (water content) beneath the surface of most permeable channel compared to other parts. In the sections below ground level outside channel which is not far from channel seen all grades $\psi$ is almost the same. This shows that the water content (water content) is almost the same in all parts of the soil depth. Furthermore, in the sections below ground level outside channel which is the farthest point from the center of the channel and below the surface impermeable channel has a value of $\psi$ is the lowest compared to another section and continued to increase to the point to its converge at certain depths. This indicates that the subsurface outside the channel in a position away from the center of the channel and below the surface of channel impermeable having a water content (water content) is the lowest compared with other parts of the ground in the domain.

4. Conclusion

Based on the discussion we concluded that DRBEM can be applied to the problem of infiltration stationary on various channel impermeable different in homogeneous soil. DRBEM applied to find the numerical solution ($\phi$) of the Helmholtz equation modified. With the numerical solution can be calculated potential suction and water content on each channel. Thus, obtained potential suction and water content on all types of channels are greatest lies in the domain below the surface in the shallow channel. Conversely, potential suction and water content on all types of channels lies in the smallest domains below ground level outside channel in the shallow part.

5. References