STABILITY ANALYSIS OF ONE PREY TWO PREDATOR MODEL WITH HOLLING TYPE III FUNCTIONAL RESPONSE AND HARVESTING

Abstract

In this paper, we discussed stability of one prey two predator with Holling type III and will harvesting at second predator population. The research aimed is, to investigate solution the predator prey model with Holling type III functional response with addition effort harvesting two predator populations and to investigate maximum profit from optimal harvesting at two predator populations. Stability of equilibrium point use linearization method and determine the stability by notice the eigenvalues of Jacoby matrix evaluation of equilibrium point and can also be determined using Hurwitz stability test by observing the coefficients of the characteristic equation. The result shows that the obtained an interior point $TE_5(x, y, z)$ which asymptotic stable according to Hurwitz stability test and find maximum profit of exploitation effort or harvest prey population and two predator populations. Predator-prey population is always exist in their life, although exploitation with efforts harvesting and given maximum profit is $\pi = 259,3999$ where to find maximum profit on critical points of surface profit function.

Keywords: Predator prey model, harvesting, equilibrium point, stability and maximum profit.

1. Introduction

The mathematical model is a part of applied mathematics, which is used to explain natural phenomena that occur, and can be used to predict the behavior of the system for a certain period of time. Mathematical modeling in the field of ecology and economy is very interesting to study because many factors that affect the lives of the population living creatures and the balance of living things and their interactions in life (Agarwal et al, 2012).

One of the mathematical models used to describe the phenomenon is predator-prey population model. The relationship between species of predator and its prey species is strong, predators will not be able to live if there is no prey. In addition, also acts as a controller predator prey populations. According Toaha (2014) those interactions between species that occur in an ecosystem can cause a state population of a species change. This interaction can have a positive impact, negative or no effect on the species that interact. One of the causes of extinction of the population is the level of predation on prey populations were extremely high and low levels of
prey or low population growth from the beginning of the prey population. Many researchers have developed a model of Lotka-Volterra by adding a few assumptions. Srinivasu et al. (2001) examine the model of Lotka-Volterra with the control system of the harvesting model and Kar (2003) examines the model of Lotka-Volterra by adding the effect of time delay on selective harvesting. Later, Zhang and Gan, (2011) examines the model of Lotka Volterra with Holling response function of type III on the interaction between predator-prey with constant harvesting effort on prey populations. Furthermore, the model still developed by Liu et al., (2012) and Wang et al., (2012) which examines concentrated effort to include the harvesting of prey populations of runaway prey (refuge). Therefore, the authors analyze the model of Lotka-Volterra with Holling response function of type III on the population of the prey and two predator populations with the assumption that both the predator population is a population which is very useful for human life, resulting in harvesting. This study aims to find out solutions predator-prey models that follow Holling type III by adding a harvesting effort on both predator populations and to determine the maximum advantage of the optimal harvesting effort in both population of prey.

2. Methodology
In general, this research framework begins with the construction of the model, which includes the stage of completion of the determination of the equilibrium point, linearized models, the analysis of the stability of the equilibrium point, and then perform numerical simulations. The variables study is to analyze the stability of the model of the two predators prey with Holling response function of type III and harvesting occurs in both populations of predators. Computing software used in this research is to use Maple.

3. Result and Findings
Population modeling of the one prey and two predator with response functions that follow Holling type III is given in equation (1) below

\[
\frac{dx}{dt} = Px - \frac{rx^2}{K} - m_1y \frac{x^2}{a_1 + x^2} - m_2z \frac{x^2}{a_2 + x^2}
\]

\[
\frac{dy}{dt} = m_1y \frac{x^2}{a_1 + x^2} - Qy - c_1yz
\]

\[
\frac{dz}{dt} = m_2z \frac{x^2}{a_2 + x^2} - Sz - c_2yz
\]

Which is \( Q = b_1 + E_1 \) and \( S = b_2 + E_2 \). Of the model (1) obtained five equilibrium points are non-negative, is \( TE_1(x,y,z), TE_2(x,y,z), TE_3(x,y,z), TE_4(x,y,z) \) and \( TE_5(x,y,z) = (x_5, y_5, z_5) \) which is \( x_5 \) the roots of \( rZ^2c_2c_1 - rKZc_2c_1 + (-m_1KSc_1 + m_1Km_2c_1 + m_2Km_1c_2 - m_2Kc_2Q + ra_1c_2c_1 + ra_2c_2c_1)Z^2 + (-rKc_1c_2c_1 - rKc_2c_2c_1)Z + (-m_1KSc_1c_1 - m_2Kc_2Qc_1 + ra_1a_2c_2c_1)c_1 \).

Equilibrium points \( TE_5 \) is a point that occurs in the first octane (interior point) if \( x_5 > 0, a_1 + x_5^2 > 0 \) and \( a_2 + x_5^2 > 0 \) namely circumstances in which the three components of the point is positive. Therefore, the stability analysis is only done at the point \( TE_5 \). From the linearization method and the Hurwitz stability test, obtained equilibrium points \( TE_5(x,y,z) \) asymptotic stable.

Numerical simulations used parameter values from other studies that are relevant with this study, namely \( K = 1000, r = 1.5, a_1 = 0.3, a_2 = 0.2, b_1 = 0.2, c_1 = 0.05, c_2 = 0.03, m_1 = 1.4, m_2 = 1.6, \) and harvesting effort \( E_1 = 1, E_2 = 0 \), in order to obtain equilibrium points (956.5085255; 39.99998834; 39.99990819) and also given \( \pi_{\text{max}} = 259.3999 \). By using linearization method obtained characteristics equation \( f(\lambda) = \lambda^3 + 4.0695\lambda^2 + 5.2721\lambda + 2.1516 \). From the characteristic equation obtained \( p_0 = 2.1516, 5.2721 = p_1, p_2 = 4.0695 \). Because \( p_0, p_1, p_2 > 0 \) and \( p_1p_2 - p_0 > 0 \) then according to Hurwitz stability test, the point \( TE_5(x^*, y^*, z^*) \) is
asymptotically stable. Figure 1, 2, and 3 below shows the behavior of the solution curve of each population against time (years) around the point of equilibrium with the initial value \( N_1(0) = 950, N_2(0) = 36, \) and \( N_3(0) = 3 \).

Figure 1. The behavior of the solution curve of prey populations \((x)\)

Figure 2. The behavior of the curve of second predator population solution \((y)\)

Figure 3. The behavior curve of the first predator populations solution \((z)\)

4. Discussion

Research shows that model of the one prey population and two predators populations prey with Holling type III functional response has five equilibrium points, one of which is the point stable interior asymptotic Hurwitz stability test and harvesting on two predator population that given maximum profit. Population model th one prey populations and two predator populations response functions that follow Holling type III is given in equation (2) below:

\[
\frac{dx}{dt} = rx \left(1 - \frac{x}{K}\right) - m_1 \frac{x^2}{a_1 + x^2} - m_2 \frac{x^2}{a_2 + x^2}
\]

\[
\frac{dy}{dt} = m_1 \frac{x^2}{a_1 + x^2} - b_1 y - c_1 y z
\]

\[
\frac{dz}{dt} = m_2 \frac{x^2}{a_2 + x^2} - b_2 z - c_2 y z
\]

An assumption that two predator population terms is very useful for human life, then the next two predator populations exploited with harvesting in each population size. With these consideration the model in equation (2) developed into,

\[
\frac{dx}{dt} = rx \left(1 - \frac{x}{K}\right) - m_1 \frac{x^2}{a_1 + x^2} - m_2 \frac{x^2}{a_2 + x^2}
\]

\[
\frac{dy}{dt} = m_1 \frac{x^2}{a_1 + x^2} - b_1 y - c_1 y z - E_1 y
\]

\[
\frac{dz}{dt} = m_2 \frac{x^2}{a_2 + x^2} - b_2 z - c_2 y z - E_2 z
\]

Explanation

\(x, y, z\): The population size of prey, the first predator and the second predator
The growth rate of the intrinsic
Harvesting efforts on the first predator and second predator populations
The rate of birth of the first predator and second predator
The constant saturation of first predator and second predator
The mortality rate of first and second predator
Measuring the rate of consumption of first predator by second predators
The first measure conversions of first predator consumed by second predator into reproductive rate of the second predator

The equilibrium point \( TE_5(x, y, z) \) equation model (1) obtained by solving \( \frac{dx}{dt} = 0, \frac{dy}{dt} = 0, \text{ and } \frac{dz}{dt} = 0 \) with linearized equation model (4) using Jacobi matrix.

\[
A = \begin{pmatrix}
\frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\
\frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\
\frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z}
\end{pmatrix}
\]

To search for Eigenvalues of matrix \( A \) measuring 3 x 3, it can be used equation \( \det(A - \lambda I) = 0 \) commonly called the characteristic equation of \( A \), namely \( f(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0. \) According to the Routh-Hurwitz stability criterion point \( TE_5 \) (\( x, y, z \)) asymptotically stable if and only if \( a_0 > 0, a_2 > 0 \) and \( a_1 > 0 \) (Toaha, 2014).

Because of the position of equilibrium point \( TE_5(x, y, z) \) in Equation (1) occurs harvesting effort imposed on the population to the assumption that \( b_1 \) + \( E_2 \) > 0 and \( b_2 \) + \( E_1 \) > 0, and by letting \( Q = b_1 + E_2 \) and \( S = b_2 + E_1 \), then \( TE_5(x, y, z) \) become point

\[
TE_5^*(x^*, y^*, z^*) = (x^*_5, m_2x^*_2S(a_1 + x^*_2)(b_1 - E_2), m_2x^*_2(a_1 + x^*_2)(b_2 - E_3))
\]

The equilibrium point \( TE_5^*(x^*, y^*, z^*) \) stable asymptotic linked to the issue of total revenue (TR), the total cost (TC) and the maximum profit (\( \pi \)). For purposes of analysis the unit price for the stock population and \( z \) population expressed as \( p_2 \) dan \( p_3 \). The total cost assumed proportional catches by harvesting effort \( E_2 \) dan \( E_3 \) with coefficients \( c_{22} \) and \( c_{33} \). According Toaha (2014) function of total revenue (TR) is expressed as follows:

\[
TR = TR(y) + TR(z) = p_2E_2y^* + p_3E_3y^*
\]  

(4)

Furthermore, substitution value \( y^* \) and \( z^* \) in equation (4) in order to obtain,

\[
TR = \left( \frac{p_2m_2x_5^2 - p_2Sb_1}{c_{2b_1}(a_1 + x_5^2)} - \frac{p_2S}{c_{2b_1}} \right) E_2^* + \left( \frac{p_3m_1x_5^2 - p_3Qb_1}{c_{1b_2}(a_1 + x_5^2)} - \frac{p_3Q}{c_{1b_2}} \right) E_3^*
\]  

(5)

According Toaha (2014) total cost function (TC) is expressed as follows

\[
TC = c_{22}E_2 + c_{33}E_3
\]  

(6)

by substituting TR value in Equation (5) and TC value in Equation (6) that will be obtained,

\[
\pi = \left( \frac{p_2m_2x_5^2 - p_2Sb_1 - c_{22}c_{2b_1}(a_1 + x_5^2)E_2}{c_{2b_1}(a_1 + x_5^2)} - \frac{p_2S}{c_{2b_1}} E_2^* \right) + \left( \frac{p_3m_1x_5^2 - p_3Qb_2 - c_{33}c_{1b_2}(a_1 + x_5^2)E_3}{c_{1b_2}(a_1 + x_5^2)} - \frac{p_3Q}{c_{1b_2}} E_3^* \right)
\]  

(7)

Because the equilibrium point \( TE_5^*(x^*, y^*, z^*) \) relies on the efforts undertaken harvesting profit function depends on the effort of harvesting. To determine the value of harvesting effort that gives the maximum benefit, it is necessary to determine the critical point of harvesting effort. Based on the equation (7), the first derivative namely obtained

\[
\frac{\partial \pi}{\partial E_2} = \frac{p_2m_2x_5^2 - p_2Sb_1 - c_{22}c_{2b_1}(a_1 + x_5^2)E_2}{c_{2b_1}(a_1 + x_5^2)} - \frac{p_2S}{c_{2b_1}} E_2^*
\]

\[
\frac{\partial \pi}{\partial E_3} = \frac{p_3m_1x_5^2 - p_3Qb_2 - c_{33}c_{1b_2}(a_1 + x_5^2)E_3}{c_{1b_2}(a_1 + x_5^2)} - \frac{p_3Q}{c_{1b_2}} E_3^*
\]  

(8)
The critical point of the equation (7) is obtained by taking equation (8) few concrete zero. Thus obtained critical point

a. From \( \frac{\partial \pi}{\partial E_2} = 0 \), obtained

\[
E_2 = \frac{p_2 m_2 x^2_2 - p_2 S b_1 - c_{22} c_2 b_1 (a_2 + x^2_2)}{2p_2 S (a_2 + x^2_2)}
\]

b. From \( \frac{\partial \pi}{\partial E_3} = 0 \), obtained

\[
E_3 = \frac{p_3 m_1 x^2_3 - p_3 Q b_2 - c_{33} c_1 b_2 (a_1 + x^2_3)}{2p_3 Q (a_1 + x^2_3)}
\]

Harvesting effort value \( E_2 \) and \( E_3 \) provide balance point \( TE_5^* (x^*, y^*, z^*) \) remains at steady state asymptotic and provide maximum benefit from the exploitation of the three populations. Functions advantage of the three populations that depend on \( E_2 \) and \( E_3 \) where the maximum profit occurs at the height of the surface of the profit function.

5. Conclusion

Model of the one prey and two predator populations with Holling type III functional responses and harvesting in two predator populations show that obtained interior point \( TE_5(x, y, z) \) that asymptotic stable according Hurwitz stability test and obtained profit maximum from exploitation effort or harvesting in two predator population. Predator-prey populations can remain stable (survive) although exploited by harvesting efforts and also provide maximum benefit is \( \pi_{max} = 259.3999 \) where obtained by the maximum profit occurs at the peak (critical point) of the surface of the profit function. For further research, it can add a variety of considerations other assumptions suppose assuming added effect of harvesting delay time and effort to see the changing dynamics of populations of organisms.

6. References


